SOLVABILITY OF THE SHELL EQUATION DERIVED BY THE Γ-CONVERGENCE

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Let us consider the boundary value problems (BVPs) of bending elastic isotropic thin media $\Omega^h : \mathcal{C} \times [-h, h]$ around a surface $\mathcal{C}$ with the Lipshitz boundary $\Gamma := \partial\mathcal{C}$, governed by the Láme equation

$$\begin{aligned} &\mathcal{L}_{\Omega^h}\mathbf{U}(x) = \mathbf{F}(x), && x \in \Omega^h := \mathcal{C} \times (-h, h),\\ &\mathbf{U}^+(\omega) = 0, && \omega \in \Gamma^h_L := \partial\mathcal{C} \times (-h, h), \qquad (1)\\ &(\mathfrak{T}(t, \nabla)\mathbf{U})^+(t, \pm h) = \mathbf{H}(t, \pm h), && t \in \mathcal{C}. \end{aligned}$$

where $\mathfrak{T}(t, \nabla)$ is the traction operator and $\mathbf{U} = (U_1, U_2, U_3)^\top$ is the displacement and *lambda*, μ are the Láme constants.

From BVP (1) we have derived the shell equation when the thickness of the layer diminishes to zero $h \to 0$, using the Γ-convergence

$$\begin{cases} \mu\left[\Delta_{\mathcal{C}}\overline{U}_\alpha + \mathcal{D}_\beta\mathcal{D}_\alpha\overline{U}_\beta - 2\mathcal{H}_{\mathcal{C}}\nu_\beta\mathcal{D}_\alpha\overline{U}_\beta - \mathcal{D}_\gamma(\nu_\alpha\nu_\beta\mathcal{D}_\gamma\overline{U}_\beta)\right] \\ +\dfrac{4\lambda\mu}{\lambda+2\mu}\left[\mathcal{D}_\alpha\mathcal{D}_\beta\overline{U}_\beta - 2\mathcal{H}_{\mathcal{C}}\nu_\alpha\mathcal{D}_\beta\overline{U}_\beta\right] = \dfrac{1}{2}F_\alpha + H^{(1)}_\alpha \quad \text{on} \quad \mathcal{C}, \\ \overline{U}_\alpha(t) = U_\alpha(t, 0) = 0 \qquad \text{on} \quad \Gamma = \partial\mathcal{C}, \quad \alpha = 1, 2, 3, \end{cases} \qquad (2)$$

provided the weak $\mathbb{L}_2$–limits

$$\lim_{h\to 0}\mathbf{F}(t, h\tau) = \mathbf{F}(t) = (F_1(t), F_2(t), F_3(t))^\top,$$

$$\lim_{h\to 0}\frac{1}{2h}\left[\mathbf{H}(t, +h) - \mathbf{H}(t, -h)\right] = \mathbf{H}^{(1)}(t) = (H_1(t), H_2(t), H_3(t))^\top$$

exist. Here $\mathcal{H}_{\mathcal{C}}(t)$ is the mean curvature of the surface, $\nu(t) = (\nu_1(t), \nu_2(t), \nu_3(t))^\top$ is the unit normal vector field on the surface at $t \in \mathcal{C}$ and $\mathcal{D}_j := \partial_j - \nu_j\mathcal{D}_4, \quad j = 1, 2, 3$ ($\mathcal{D}_4 = \partial_\nu = \sum_{k=1}^{3}\nu_k\partial_k$) are the Günter's tangential derivatives.

The object of the investigation is the solvability properties of the obtained equation of shell (1). We have proved the following.

THEOREM. *The operator in the shell equation* (2) *is positive definite and the boundary value problem* (2) *has a unique solution in the classical setting:*

$$\overline{\mathbf{U}} := (\overline{U}_1, \overline{U}_2, \overline{U}_3)^\top \in \mathbb{H}^1(\mathcal{C}), \qquad \frac{1}{2}\mathbf{F} + \mathbf{H}^{(1)} \in \mathbb{L}_2(\mathcal{C}).$$

The investigation is carried out in collaboration with T. Buchukuri, (Tbilisi).