Stochastic integration in various function classes – algorithms and complexity

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This work was inspired by a paper of Eisenmann and Kruse [1] and motivated by recent research on stochastic computation with discontinuous input data. We study algorithms for and the complexity of stochastic integration with respect to the Wiener sheet measure $\int_{[0,1]^d} f(t)dW_t$ of stochastic functions f with the following types of regularity: f is assumed to belong to $L_u(\Omega, X)$, where $1 \le u < \infty$ and X is

- a Sobolev space $X = W_p^r([0,1]^d)$ $(r \in \mathbb{N}, 1 \le p \le \infty)$, or
- a Besov space $X = B_{pp}^r([0,1]^d)$ $(r \in \mathbb{R}, 0 < r < \infty, 1 \le p \le \infty)$ (also called Sobolev-Slobodeckij space), or
- a Bessel-potential space $X = H_p^r([0,1]^d)$ $(r \in \mathbb{R}, 0 < r < \infty, 1 < p < \infty)$.

In all cases it is assumed that $r/d > 1/p - 1/2$. Thus, the integrands may be discontinuous in (multidimensional) time. Information about f that can be used by the algorithms consists of function values while that about W_t may be function values or scalar products with polynomials of a given degree. Both deterministic and randomized algorithms are considered. We present and analyze algorithms of optimal order and prove matching lower bounds, thus determining the complexity of the problem. This extends results from [1], where algorithms for the one-dimensional case $X = B_{pp}^r([0,1])$ with $0 < r < 2$, $2 \le p < \infty$ were established, and from [2], where only deterministic integrands were considered.

The needed concepts on stochastic integration, function spaces, and complexity theory will shortly be reviewed, as well.

References

[1] Monika Eisenmann, Raphael Kruse. Two quadrature rules for stochastic Itô-integrals with fractional Sobolev regularity. arXiv:1712.08152

[2] S. Heinrich, Complexity of stochastic integration in Sobolev classes, J. Math. Anal. Appl. 476 (2019), 177–195.